Whitepaper - Estimating Bicycle Tire Friction Coefficient

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5/23/2020

Introduction

This document is meant to serve both as an introduction to the physics of the coefficient of friction of tires, and to lay of the modeling used in our integrated bicycle tire coeffecent of friction augmented reality demo.

The origin of the phrase "this is where the rubber meets the road" is that the tire-ground interface is central of all vehicle performance and safety - and a modern motor vehicle's running gear (wheels, suspension, steering, powertrain & chassis) is in great part designed so as to optimize the performance and longevity of tires. The physics and technology of a bicycle's running gear are comparatively simple - no need to consider the behavior of weight transfer left to right during turns, or (for modern vehicles) the effects of anti-lock brakes or limited slip-dfferential drive trains.

However, the underlying tire physics (especially during braking) are quite similar. The ability to brake quickly when necessary is a function of the coefficient of friction of the tire-ground interface which we will describle below on our way to developing a kinematic model for estimating the sliding coefficient of friction (COF) of tires. In our demo video for this, we implement this estimator using Sensar's Starstream technology, which allows for rapid protytping and development of IoT systems integrated with Augmented Reality (AR).

Coefficent of Friction

In it's simplest form, the coefficient of friction is the ratio of the force we must apply to an object to move it across a given surface, to the weight of that object. Note - in automotive applications, additional considerations such the aerodynamic downforce created by the vehicles movement through air much be considered, but will be neglected here.

We know that force is mass times acceleration ($F = ma$), and an object's weight - which is a force - is the object's mass times the earth's (average) gravitational acceleration constant (g), which is $\sim$ 9.81 m/s^2 (meters per second per second, where the m for meters is not to be confused with the mass m.

Conversely, $a = \frac{F}{m}$, so that when you apply a force to a mass it imparts an acceleration which is directly measurable using sensors such as accelerometers.

As seen in Figure 1, if the coefficient of friction is equal to ma divided by mg, then the m's cancel and the coefficient of friction is just the acceleration divided by the constant g. The coefficient of friction is then, of course, a unitless number.

However, for this classic example of a sliding block, there are actually two kinds of friction - static and dynamic (see Figure 2 below).

In Figure 2, the force is stated in SI unit of Newtons (N). The static frictional resistance (stiction) turns into kinematic frictional resistance once the block moves. It is the general case that the constant force that must be applied to maintain a constant velocity is less than can be applied in stiction - and thus the sliding COF is less than the stiction COF.

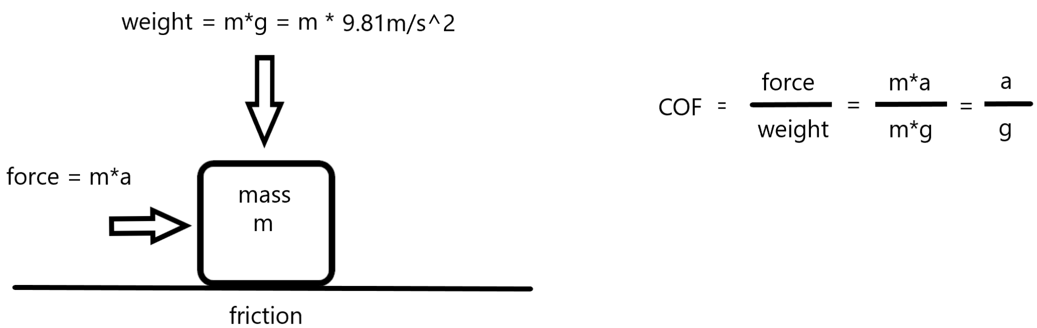

Figure 1: Sliding Block

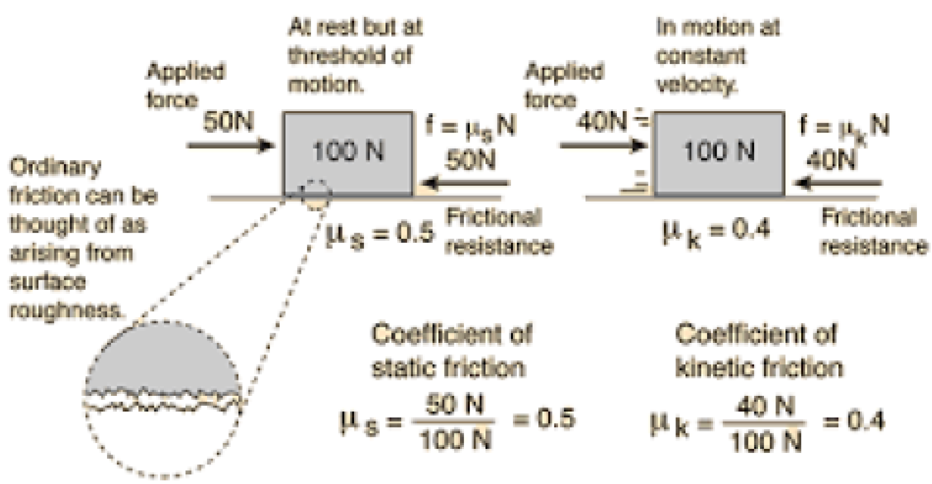

Figure 2: Stiction and Friction

Tires and Coefficient of Friction

Tire Types

Modern pneumatic automotive tires come in many flavors, but for passenger automobiles it generally comes down to radial and bias-ply tires. In general, radial tires have significant advantages, and have mostly won out, because they improve fuel consumption by reducing rolling resistance (energy dissapated from tire-flexing), provide a softer ride with less vibration, and extended tire life. However, this does come with a few relative penalties: low lateral stiffness that may cause causes the tire sway as the speed of the vehicle increases, increased vulnerability to abuse when overloaded or under-inflated, and higher initial cost (that is outweighed by the longer lifespan).

Conversely, for bicycle tires, the bias ply tire has the **advantages** decreased vulnerability to abuse when overloaded or under-inflated, and lower initial cost (where the lifespan issue is less of a consideration because bicycle tires usually see lower mileage per unit time).

The Tire-Ground Interface

For wheeled vehicles, the object that moves is the tire, and the surface it moves along is the ground. The modern pneumatic (air filled) tire is really a bit of a wonder, with the weight force of the vehicle ultimately being opposed by a flat spot on the bottom of the tire that is called the contact patch. The pressure of the air in the vehicle's tires - which might be measured in pounds per square inch - times the area of the contact patches - which might be measured in square inches - must equal the full weight of the vehicle. So the whole weight of the vehicle is felt these small contact patches, with the matching patches of ground underneath they supplying the opposing upward force. One might consider these intestices as the frictional system considered above - however, the tire is of course rolling, which provides extra complications

Tractive Force vs Slip ratio

The tractive effort of the tire is just the force applied by the tire against the ground along the direction that the tire is pointing. During breaking, this force is opposed to the direction that the vehicle is moving. Note that this force is equivalent to providing an acceleration of the vehicles mass, and since velocity is the integral of acceleration, the vehicle slows over time due to the braking force.
When a driving torque is applied to a pneumatic tire, a tractive force is developed at the tire-ground contact patch, see Figure 3. At the same time, the tire tread ahead of and within the contact patch is subjected to compression. A corresponding shear deformation of the sidewall of the tire is also developed.
Then as tread elements are compressed before entering the contact region, the distance that the tire travels when subject to a driving torque will be less than that in free rolling. This phenomenon is usually referred to as longitudinal slip, though the definition of slip - that the speed of the tire where it interfaces with the ground is different than the linear speed of the vehicle - also extends all the way to the phenomenon where the tire spinning fast enough to spin freely (in acceleration) or is locked and not spinning at all (in braking).

The tractive force of the tire is proportional to the applied wheel torque under steady-state conditions, see Figure 4. When applying tractive force, at first the wheel torque and tractive force increase linearly with slip because in this regime slip is mainly due to elastic deformation of the tire tread. This corresponds to section OA of the curve shown in Figure 4.

Further increasing wheel torque and tractive force results in part of the tire tread sliding on the ground. Under these circumstances, the relationship between the tractive force and the slip is nonlinear. This corresponds to section AB of the curve shown in Figure 4. Experimentally, the maximum tractive force of a pneumatic tire on hard surfaces is usually reached somewhere between 15 and 20% slip. Any further increase of slip beyond that results in an unstable condition, with the tractive effort falling rapidly from the peak value to the pure sliding value. (Wong (2008)).

This is why, when a dragster applies too much force, the tires quickly spin freely (much faster that the vehicle's forward velocity) due to a cascade of significantly lower COFs.

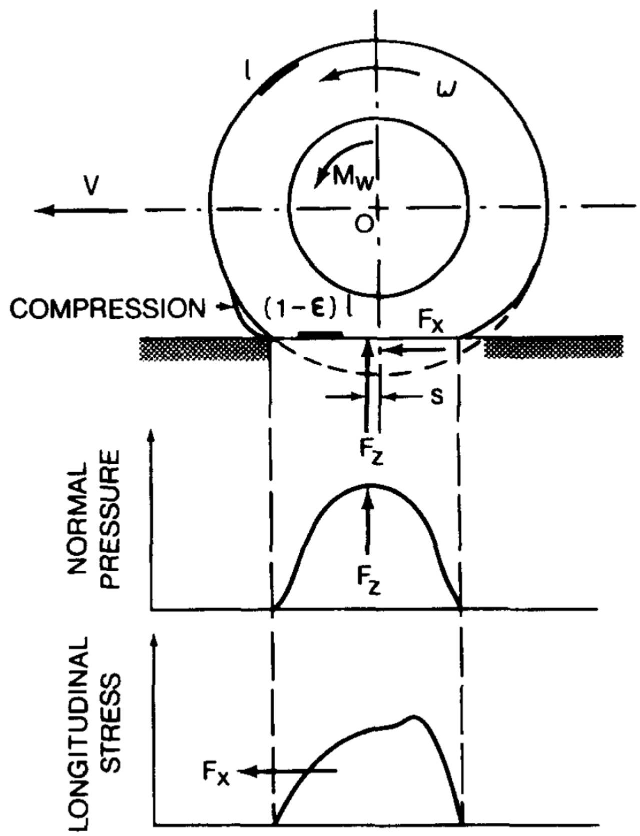

Fig. 1.15 Behavior of a tire under the action of a driving torque. (Reproduced with permission from *Mechanics of Pneumatic Tires,* edited by S.K. Clark, Monograph 122, National Bureau of Standards, 1971.)

Figure 3: Contact Patch (FIGURE 1-15 Wong (2008))

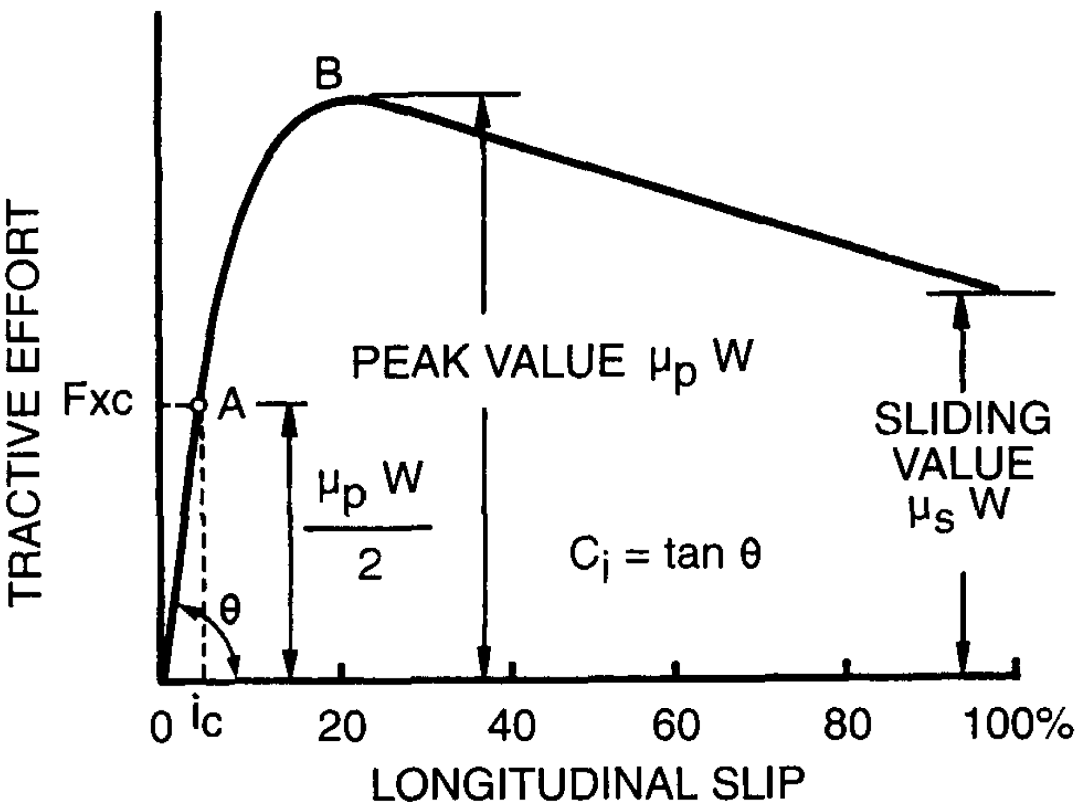

Figure 4: Longitudinal Slip vs Tractive Effort (Wong (2008))

As a corollary, when a **braking** tractive force is applied, the tread elements are compressed **after** entering the contact region. But the tractive effort curve is similar, and "locked" brakes (i.e. the wheels have stopped turning so that the logitudinal slip is 100%) are similarly created in a cascade of significantly lower COFs, meaning less decceleration and longer times to actually stop,

The real-world concerns about slip and traction extend to both longitudinal and lateral slip (slip of the tire sideways from the direction it is pointed), as both types of slip occur as a vehicle corners - which we will also neglect in this analysis.

Kinematic Bicycle COF Estimator

Estimating the Coefficient of Friction

As you might imagine, measuring the *time-varying* coefficient of friction of a moving bicycle and the (perhaps significantly varying condions of the) ground it moves over is complicated. In order to estimate the time-varying actual coefficient of friction (for just one type of road and set of environment), some measuremeny of the system during the necessary range "excitations" of the system - accelerations and velocities of the vehicle and tires with respect to road - is of course required. For motor vehicles, there has been quite a bit of research done here (examples) - with proper modelling (including a model of basically everything about the running gear of the motor vehicle) and system identification (statistical methods used to estimate accurately various parameters of the system that are hard to measure independently), the results can be excellent. We implement such as system for our motor vehicle COF estimator.

However, to remove the complexity of the modeling and system identification steps for discussion here, and by way of demonstration of the utility of Starstream for quick sensed-system investigation and development integrated with AR, we can use a very simple kinematic model to make estimates of the **sliding** COF of bicycle tires simply by exciting the system agressively (by slamming on the brakes) and measuring the acceleration of the vehicle once the tires lock. This provides an estimate of the **minimum** COF available to bicycle on whatever surface the test is run on, which is an important parameter.

A kinematic model uses only known initial conditions of position, velocity and/or acceleration of points within the system to predict behavior of unknown (and future) conditions of the system.

Dynamic of Moving Bicycle with Non-Rotating Tires

This is the scenario when we slam on the brakes **assuming the brakes immediately lock up and the wheel stops rotating**. Under these assumptions the sliding Coefficient of Frictional is easily calculated using a kinematic model.

Coefficient of Friction and Applied Force

The general equation for dynamic friction is:

$F_d = \mu_d N$

where:

- F_d is the resistive force of locked-tire friction (decelleration the vehicle!)
- μ_d is the coefficient of locked-tire friction for the two surfaces (Greek letter "mu" sub d)
- N is the normal force pushing the wheel to the surface (i.e. the weight of the vehicle)

Remembering that $F = ma$, and that the mass of the vehicle is the same for both weight and acceleration/decceleration, then

$\mu_d = F_d/N = a_d\ m/a_g\ m = a_d/a_g$,

where a_g is the gravitational acceleration constant of 9.81m/s^2.

Noting that If we measure acceleration/deceleration in units of "G's" rather than m/s^2, then the COF is

$\mu_d = a_d$ (minus the units)

Of course, to observe the tractive force vs slip curve results, we would need an estimate of the slip (and a dynamic model) as well. However, estimating the 100% slip point - the fully-sliding friction - can be done straightforwardly by determining when the wheel stops spinning and then using

$\mu = a_d/a_g$.

We also require a way to directly measure the a_d. One method would be to use a full Inertial Measurement Unit (IMU) to keep track of the orientation of the bicycle in space relative to velocity vectors and direction of gravity.

Locked-Wheel, Straight-Line Deceleration COF Estimate

A much simpler approximation is to 1) simply measure the **scalar** magnitude of the accelerations on the bicycle to estimate the COF, as well as 2) effectively model the system as a single, sliding block.

The maginitude of the the acceleration can be found by taking the square root of the sum of the squares of the components. This operation is often called quadrature addition (and is basically a multi-dimensional version of pythagorus's rule for finding the length of the hypotenuse).

These approximations help us out quite a bit, as we don't have to worry about how much the bike leans forward during sudden braking, or which of the tires is doing the most breaking, etc. In fact, we can actually do away with the issue of the *orientation* of both the bicycle and the acceleration sensor completely, since we know that the over all three measurement axes, *regardless of orientation*, the scalar quadrature acceleration

$a_{quad} = \sqrt{x_{acc}^2 + y_{acc}^2 + z_{acc}^2}$

will be the quadrature sum of $1g$ from gravity plus the vehicles decceleration, *and* these two forces will be orthogonal to each other. Thus the decceleration will be the measured $a_{quad} = \sqrt{1^2 + a_d^2}$, and thus:

$a_d = \sqrt{a_{quad}^2 - 1}$

However, clearly this simple kinematic model won't work anything other than straight-line decceleration with locked-up wheels.

Similarly, we look for a simplifying assumption about the angular velocities of the bicycle's tires and the gyro sensor used to sense them.

The bicycle's tires' quadrature angular velocity is

$\omega_{quad} = \sqrt{x_\omega^2 + y_\omega^2 + z_\omega^2}$.

Note that as long as the bike does not lean left or right (too much), the quature angular velocity is just the angular velocity of the tire around the hub, and since $\omega * \text{radius}_{tire} \sim=$ bicycle velocity, then bicycle velocity $= \sim 0$ when $\omega =\sim 0$.

Implementation of the Estimator

This method, of course, introduces many sources of error. Any calibration errors in the value used for gravity will become systematic error, taking a quadrature sum of three measurements that have error results in larger error (both random and biased),and things like vertical mechanical noise from bumps in the road get turned into "signal" (generally biased the approximation of a_d upwards) that may even (occaisionally) exceed 2 G's (and thus exceed the physically plausible).

These latter two issue requires that signal below and above some critical thresholds be rejected - say above $0.9g$ and also below some average value of mechanical+sensor noise that can be determined emperically when the wheels are rotation. But with careful averaging of signal, an estimate is possible.

Then the easiest validating model we can use to calculate COF is to:
1. Have a vehicle that does not have antilock brakes so we can "lock" the tires (like a bicycle, or old an car).

2. Simple Calibrations. Calculate quadrature gyro measurements while stopped, and use the three standard deviation value as $\omega \sim 0$. Calculate quadrature accelerometer measurements while riding at nominal speed, and use three-standard deviation accelerations as the noise floor to ignore for estimating $a_d \sim 0$.
3. Detect (via tire gyro measurement of angular velocity ω) that the wheels have "locked", i.e. have a sensed angular velocity ω of ~ 0 degrees per second (dps), where approximately here means within lower than several standard deviations of the noise of the measurement.
4. Sense at that moment (via the vehicle body accelerometer) the decceleration of the vehicle a_d in m/s^2 and divide by 9.81 m/s^2 (or better yet, just measure in G's), giving us a measurement of μ.
5. Repeat this measurement of μ_d for as long as we both get measurements of $a_d >\sim 0$ (as we are done sliding when $a_d = 0$ and $\omega =\sim 0$, and *average the results* to get a good estimate. 6. Sum up all the measurements of a_d when $\omega >\sim 0$ during the braking event - this is μ_d.

Model **When** the quadrature omega measurement
$\omega_{quad} = \sqrt{\omega_x^2 + \omega_y^2 + \omega_z^2} < \epsilon$,

where $\epsilon \; = 3$ std's of ω

then we might think that

$a_d = \sqrt{a_{quad}^2 - 1}$ (regardless of orientation).

where

$a_{quad} = \sqrt{x_{acc}^2 + y_{acc}^2 + z_{acc}^2}$.

One, this clearly *isn't* true for anything other than straight-line decceleration with locked-up wheels. However, we also have a real difficulty with this formulation - it is clearly possible (due to measurement error and also mechanical noise as the bicycle runs over bumps in the road) to encounter

$a_{quad} = \sqrt{x_{acc}^2 + y_{acc}^2 + z_{acc}^2} < 1$

which would make a_{quad} imaginary, so we instead define

$a_d = \sqrt{|a_{quad}^2 - 1|}$

and note that for any reasonable coefficient of friction this will not matter.

Then, we have simply:
$\mu_d = F_d/N = m \; a_d/N = m$ (car mass in kg) a_d (m/s^2) / (m(car mass in kg) 9.81m/s^2) = $a_d/9.81$

Sampling rates We could of course do this experiment many times over, as long as we get one measurement of μ_d per experiment and repeat the results. But we actually really want to get a "good" estimate of COF μ_d from one braking event. So let's look at the accuracy of our estimate under different assumptions.

Samples over the Deceleration increment The equation for how long it takes to stop the vehicle is:
$t_0 = (v_i - v_0)/a_d = (v_i - v_0)/(F_d/m)$

where
t_0 = time to get to v_0 (time to stop)
v_i = initial speed
v_0 = end speed (here 0)
a_d = decceleration
F_d = decceleration force
$m =$ mass of vehicle (here)

We know / can calculate all of these parameters. By example *if* the COF is ~0.7, the bicycle will experience a decceleration force a_d of $0.7 * 9.81 m/s^2 = 6.867 m/s^2$. Let's assume the bike starts breaking at are 30mph $= 13.4112 m/s$, then:

$t_0 = (v_i - v_0)/(a_d) = (13.4112m/s)/6.872m/s^2 = (13.4112m/s)/(6.872/s^2) = 1.951521188s.$

At a 5 hz sampling rate, we would then get $5/s * 1.95s = 9.75$ (round to 9) measurements per stop. We get a better accuracy by averaging as error = error / N. If the errors of each measurement are independent then the from statistics the Standard error of a mean of nine measurements is

$\sigma_{\bar{x}} = \sigma_x/\sqrt{N} = \sigma_x/3.$

The error of the quadrature acceleration measurement is

$a_{d\ err} = \sqrt{x^2_{acc\ err} + y^2_{acc\ err} + z^2_{acc\ err}}$

and since the measurment errors are identical and independent

$a_{d\ err} = \sqrt{3}\ x_{acc\ err}$

so that

$\sigma_{\bar{d}} = \sigma_{x\ acc}/\sqrt{3}.$

However, because we are introducing mechanical vibration in the up-down direction (which experimentally we find to be much larger than the random error measurement of the acceleromters), we are forced to to determine the error on the acceleration experimentally.

Three Standard Deviation (STD) of errors on Accelerometer: Ideally, we want to reject any measurement of the acceleration which is less than three standard deviation of the noise (which will consist mostly of mechanical noise). Since a_d is always a positive scalor,the errors will be centered around some positive value as well - we are not deeply concerned about the distribution of the errors, simply determining the the line that 99.7 percent of the a_d measurements fall beneath when moving at speed (without breaking) should be sufficient.

Three standard deviations of the (assumed random and uncorrelated) errors on the accelerometer measurement at 5hz was measured experimentally to be approximately $0.31g$.

Three STD Errors on Gyro measurement: The gyro measurement does not suffer from the same significant mechanical noise issues as the acceleration measurements.

The standard deviation of the (assumed random and uncorrelated) errors on the gyro at 5hz can be measured experimentally to be approximately 0.100 dps, which is in line with three standard deviation of the quadrature gyro instrument error noise. Note that we don't actually care what the tire angular velocity, radius, linear speed, or slip ratio is - just that the the tire has stopped spinning (which will be true 99.7% of the time when $\omega_{quad} < 0.100$)

Example Results

Figure 5 shows the results from the gyro and accelerometer quadrature measurement for a typical test run. There are three "events" - accelerations to speed, followed by "locked-brake" stops. The locked-brake stops occur at approximately 450 seconds, 975 seconds, and 1125 seconds. Note that there are large accelerations as the bicycle is brought up to speed due to pedal effort and other mechanical noise.

When the estimation conditions are met - the wheel is not spinning **and** there are accelerations larger than the the three standard deviations level of noise (0.31 $g's$), we get estimates. In Figure 6 we have averaged the data within a window so that the estimated values are clearer for the plot. The estimated values vary both inside and between the braking events because the road surface under the tires is different. The precipitous drop in the COF at the end of the third braking event is due to encountering sand on the road after braking has begun.

Bibliography

Wong, J. Y. 2008. *Theory of Ground Vehicles*. 4th ed. Wiley.

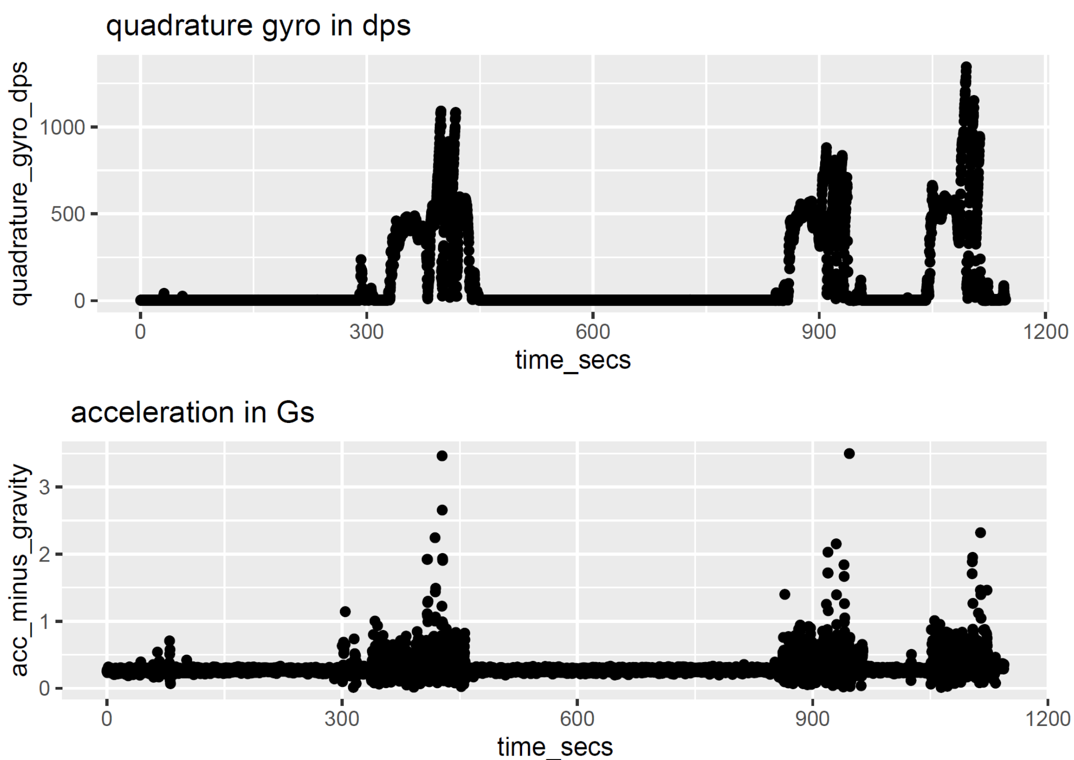

Figure 5: Bicycle Sensor (Quadrature) Measurements

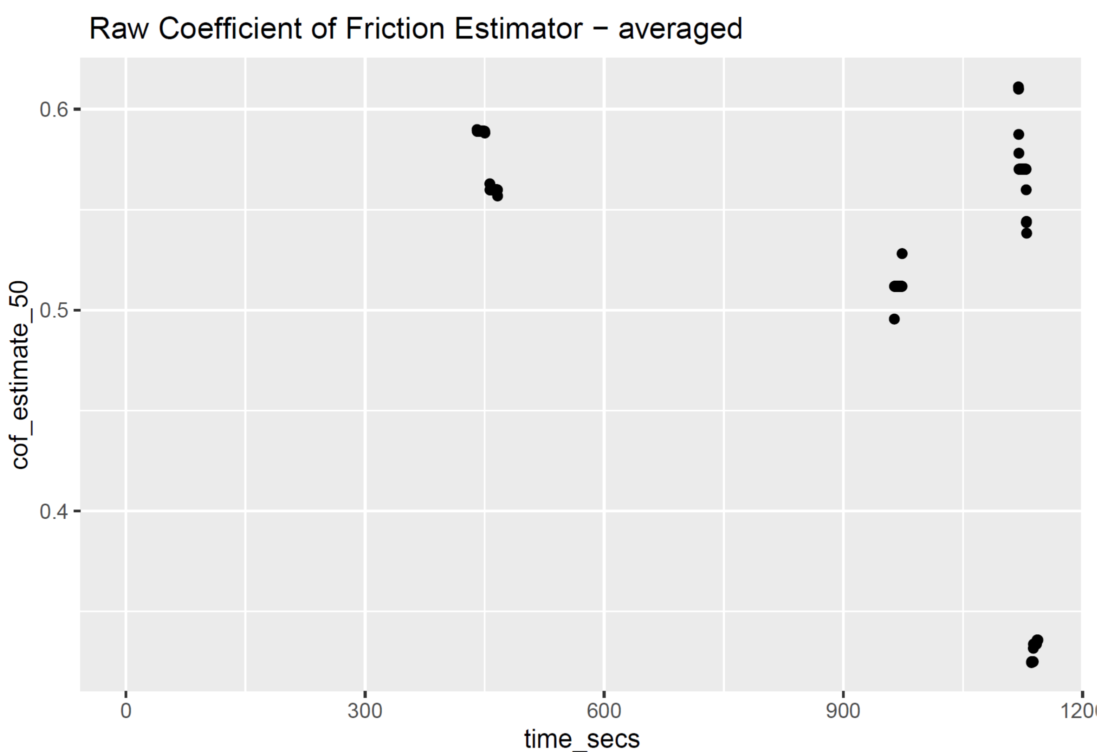

Figure 6: Coefficient of Friction Estimates